S. A. Computer Olympiad

Second Round Open Div. 2006

For Grades 11, 12 and all wishing to qualify for the Third Round

Q1. Decompression

Prepared by Donald Cook

Description

To save time in the transmission of data, it is often compressed. One simple way of doing this is to encode lengthy sequences of the same value by using a single instance of that value followed by the number of times it is repeated.
For example if you have a sequence 2222227777 it would be represented as 2 6 7 4.

Task

Your task is to write a program that will receive as input a sequence of numbers representing encoded data and decompress it. That is, print out the original sequence in its long form.

Constraints

The input will be terminated by -1.
An expanded sequence will only contain the digits from 0 to 9 and will not be longer than 50 digits.

Sample run

Note the input is terminated by -1.

Input

Enter the string to decompress:
0 14 1 7 2 18 **-**1

Output

000000000000001111111222222222222222222

Test your program with

a. 0 5 1 10 2 15 3 10 -1
b. 1 3 3 5 2 7 5 9 8 11 -1
c. 2 5 4 5 8 15 4 5 2 5 -1

Q2. Sets

Prepared by Marco Gallotta

Description

A set in mathematics is a collection of distinct elements. The union of two sets A and B consists of all unique elements which are either in set A, or set B, or both set A and set B. The intersection of two sets A and B consists of all elements which are in both set A and set B.

Task

Write a program that reads in two sets of integers, A and B, and calculates both the union and intersection of A and B. The new sets must be output in ascending order, i.e. smallest integers first.

Constraints

- Size of set A and B ≤ 20
- An element will not be repeated in a set.

Sample run

Suppose A = {1, 2, 4} and B = {2, 4, 6, 8}. The union of A and B is the set {1, 2, 4, 6, 8}, since all the numbers appear in *at least one of* A or B. The intersection of A and B is the set {2, 4}, since they are the only numbers that appear in *both* A and B.

Input

Enter the size of set A: 3
Enter element 1 of set A: 1
Enter element 2 of set A: 2
Enter element 3 of set A: 4

Enter the size of set B: 4
Enter element 1 of set B: 2
Enter element 2 of set B: 4
Enter element 3 of set B: 6
Enter element 4 of set B: 8

Output

A union B: 1 2 4 6 8
A intersect B: 2 4

Test your program with

a. A = {1, 2, 3, 4, 5, 6, 7, 8}
B = {1, 4, 9, 16, 25}
b. A = {7, 3, 16, 11, 8, 5, 19, 20, 1, 25, 15, 30}
B = {18, 4, 26, 3, 16, 21, 5, 19, 2, 9, 27, 32, 14, 6}
c. A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
B = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

Q3. Reduce

Prepared by Marco Gallotta

Description

Every letter in the English language can be represented as a number. Consider the numeric representation of a lower case letter to be equal to its position in the alphabet with 'a' being represented as 1 and 'z' being represented as 26.

Task

Given a word, your task is to create a new word by splitting the original word into pairs of adjacent letters and then combining these as follows. You split the original word into pairs by starting at the beginning of

the word, and grouping each successive pair of letters as you read from left to right.
If the numeric representations of a pair are A and B, then:

1. If A + B ≤ 26, then add the letter whose numeric representation is A + B to the new word
2. If A + B > 26: if A < B, add the letter whose numeric representation is (B – A) + 1 to the new word; otherwise add the letter whose numeric representation is (A – B) + 1 to the new word.

If your original word has an odd number of letters, then at the end you will have one spare letter which is not in a pair: add this letter to the end of the new word.

You should then repeat the above process until you are left with a word that is a single letter. You must also print out the word at each cycle in this process.

Constraints

- The length of the word ≤ 12.
- The word will consist of lower case letters only.

Sample run

Consider the word "hello", which can be represented by the numerical values {8,5,12,12,15}. After one iteration you should get a numerical representation of {[8+5], [12+12],15} = {13,24,15} = "mxo".
A second iteration and you get {[(24-13) +1],15} = {12,15} = "lo", followed by a third giving you {(15-12)+1} = {4} = "d".
Now that we are left with only a single letter, we are done.

Input

Enter word: hello

Output

hello
mxo
lo
d

Test your program with

a. reduce
b. computer
c. shuttleworth

Q4. Tasks

Prepared by Timothy Stranex

Description

Jim, a very busy person, has *N* tasks he has to complete. Some of these tasks must be performed before others so he must work out an order in which to do them. The tasks are numbered from 1 to *N*. There are *M* task dependencies which are numbered from 1 to *M*. A dependency (*a b*) indicates that task *a* must be performed before task *b*.

Task

Write a program that asks the user to enter *N*, *M* and the dependencies and then outputs all possible task orderings. Sometimes it will be impossible to find any orderings. In this case, print out "Impossible".

Constraints

$1 \le N \le 7$
$1 \le M \le 10$

Sample run 1

Input

Enter N: 4
Enter M: 3
Enter dependency 1: 1 2
Enter dependency 2: 4 3
Enter dependency 3: 1 3

Output

1 2 4 3
1 4 2 3
1 4 3 2
4 1 2 3
4 1 3 2

Sample run 2

Input

Enter N: 2
Enter M: 2
Enter dependency 1: 1 2
Enter dependency 1: 2 1

Output

Impossible

Test your program with

a. N=5, M=4, (5 1), (1 2), (2 3), (3 4)
b. N=6, M=5, (3 4), (5 6), (6 3), (1 2), (4 5)
c. N=5, M=5, (1 5), (2 5), (3 5), (5 4), (1 2)

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Q5. Ladders

Prepared by Nick Pilkington

Description

Ladders is a game played on a board with N sequentially numbered squares (1 to N). A player starts on square 1, and then moves forward according to the value rolled on a 6-sided die at each turn. There are ladders linking some pairs of squares. If a player ends his turn on a square that has a ladder beginning at that same square they are immediately transported to the end of that ladder. Having whiled away many cold winter evenings playing Ladders you have acquired the uncanny skill of being able to roll any number (1,2,3,4,5,6) on the die at each turn. Given the size of the board and the positions of all ladders on the board, find the minimum number of turns in which you can reach the last square on the board and so finish the game.

Task

Find the minimum number of 6 sided die rolls it will take to reach square N from square 1.

Constraints

The board will have N squares. $1 \leq N \leq 10000$
There will be M ladders. $0 \leq M \leq 100$. A ladder is defined as two numbers, A and B. A indicates the start of the ladder and B indicates the end; B > A. No ladder can start on square 1, and no two ladders will start or end on a common square.

Sample run

Input

Enter N: 14
Enter M: 2

Enter start and end square 1: 2 6
Enter start and end square 2: 7 8

Output

2

Test your program with

a. N=9876, M=0
b. N=100, M=2 (15,35), (17,80)
c. N=10000, M=7 (20,40), (25,100), (45,105), (1002,1451), (1003,1281), (1282,1345), (1346,1726)